

## SIMULTANEOUS PROPAGATION OF BOTH BOUND AND LEAKY DOMINANT MODES ON CONDUCTOR-BACKED COPLANAR STRIPS

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### Abstract

Contrary to expectations, we found recently that a *new leaky dominant mode* may be present on conductor-backed coplanar strips at the same time as the conventional bound dominant mode. The frequency range over which both these modes can propagate *simultaneously* increases as the strips become wider, and can be quite large even for fairly narrow strips. These new features, which could affect circuit performance significantly, have been derived theoretically and verified by measurements.

### I. INTRODUCTION

Guided modes on most open guiding structures are purely bound in one frequency range and leaky in another. It is known that such modes exhibit a "spectral gap" when a guided wave purely bound to the structure changes into a radiating leaky wave as the frequency is raised past some critical value. In fact, our previous work[1] has shown that the behavior within the various spectral gaps usually forms a common pattern for most printed-circuit lines, including coplanar strips, slot line, coplanar waveguide, and microstrip line on an anisotropic substrate. A typical spectral gap involves a *short frequency range* in which, at the lower frequency end, the mode remains purely real but becomes improper (nonspectral), meaning that the fields are no longer bound but increase to infinity transversely, and, at the upper frequency end, the mode becomes complex and leaky, but is nonphysical. We have also shown in [2] and elsewhere that the solution in these narrow frequency ranges within the spectral gap is not captured in a steepest-descent-plane plot, meaning that the solution there is nonphysical and does not contribute to the total field.

Thus, for guiding structures with a common (or regular) spectral-gap pattern, the bound dominant mode can be observed only *below* the frequency at which the spectral gap sets in, and the leaky dominant mode propagates only *above* the frequency at which the spectral gap ends. This means that the propagation range in frequency for the bound dominant mode is *distinctly separated* from that for the leaky dominant mode by the spectral gap.

For conductor-backed coplanar strips (see Fig.1 for its cross section), we have found recently that the usual, or regular, spectral gap occurs only for relatively narrow strip widths ( $w/h$

= 0.25, typically, for the geometry chosen). As the strips are made somewhat wider, the nature of the spectral gap changes, and a new complex solution emerges, although it is nonphysical (not captured in the steepest-descent plane). As the strips are made still wider, though still not very wide ( $w/h = 0.50$  in our case), a surprising effect occurs: The spectral gap disappears, and instead a frequency range appears within which the bound and leaky dominant modes exist *simultaneously*, and are *both physical*. Increasing the strip widths further produces a rapid increase in the frequency range over which both solutions are present. In fact, for a relatively small further increase, to  $w/h = 0.70$ , the frequency range over which the bound and leaky solutions overlap is already greater than two to one.

We should recognize that leakage effects occur when the phase constant of the dominant mode becomes smaller than that for the surface wave on the surrounding substrate, which in this case is a grounded dielectric layer. Thus, the dielectric layer must be electrically about a quarter-wavelength thick or so, which means that this new and interesting effect will likely arise only in the millimeter-wavelength range for typical conductor-backed coplanar strips. On the other hand, if the strips are made wider than usual, the low-frequency end of the overlap range drops substantially, and the effect will be noted at *much lower* frequencies.

The significance of this discovery regarding the simultaneous presence of the two modes is that a source intended to excite the bound mode will also excite the leaky mode with roughly the same amplitude. That will happen because both modes are "*dominant*" modes, in the sense that the current distributions on the strips are essentially the same for both modes, and that the fields of both modes are very similar in the central guiding region. The latter statement will be verified by

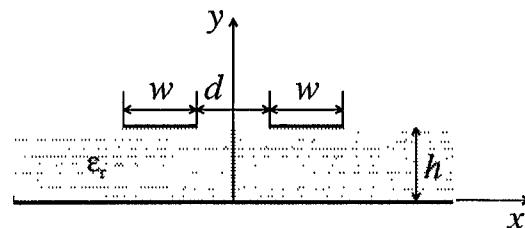


Fig.1. Cross section of conductor-backed coplanar strips, showing the notation employed.

vector electric field plots presented during the talk. The simultaneous presence of the two modes will almost certainly produce design problems that require further investigation.

We discuss the new behavioral features first in terms of the usual normalized wavenumber *vs.* frequency plots, where we examine the evolution of the bound and leaky solutions as the coplanar strips are slowly made wider. Then we present the results of measurements that we took that confirm directly and quantitatively the simultaneous presence of the bound and leaky dominant modes.

## II. EVOLUTION OF THE NEW SOLUTIONS FOR CONDUCTOR-BACKED COPLANAR STRIPS

When the coplanar strips have  $w/h=d/h=0.25$ , with  $\epsilon_r=2.25$ , where the dimensions are defined in Fig.1, the behavioral features of the transition from the bound dominant mode at lower frequencies to the leaky dominant mode at higher frequencies are the usual, or regular, ones. The variations for that case of the phase constant  $\beta$  and the leakage constant  $\alpha$ , both normalized to the free-space wavenumber  $k_0$ , as a function of  $h/\lambda_0$ , the thickness of the dielectric substrate relative to the free-space wavelength, are shown in Fig.2. The abscissa axis actually represents the variation with frequency, since  $h$  was held constant in the calculations.

The spectral gap, which can barely be seen in Fig.2, is shown in Fig.3 on a greatly enlarged scale. On the lower left in Fig.3, we follow upward the proper, or spectral, real (bound) solution for the coplanar-strips' dominant mode. As the frequency increases, this solution reaches the dispersion curve of the  $TM_0$  surface wave at the point ①, and then continues as an improper real solution (dashed curve). At the frequency indicated by ②, the improper real solution curves back, but the improper complex (leaky) solution, shown solid, then moves to the right, to higher frequencies. Since the improper solutions above the  $TM_0$  surface-wave curve are not captured in the steepest-descent-plane plot and are not physically meaningful, we see that we have a spectral gap of the usual type between points ① and ③.

We have found that for conductor-backed coplanar strips the pattern of the spectral gap, and even if there is one, depends strongly on the dimensional parameters. The changes that occur when the relative strip width  $w/h$  is varied, while keeping  $d/h$  and  $\epsilon_r$  the same, are indicated clearly in Figs.4(a) and (b). In Fig.4(a), the value of  $w/h$  has been increased from 0.25, in Figs.2 and 3, to 0.375, while keeping the other parameters unchanged. There is still a spectral gap in this case, but a **new feature** arises in that the improper real solution undergoes a change in curvature at point ②, and that at this point an additional (new) improper complex (leaky) solution is created, which moves to the left, to **lower** frequencies. This new leaky solution is not physical, however, since it lies above the  $TM_0$  surface wave curve and is not captured in the steepest-descent plane.

When the value of  $w/h$  is increased slightly further, point ② moves down until it meets point ③ in Fig.3 and in Fig.4(a), at which time the new leaky solution becomes **continuous** with

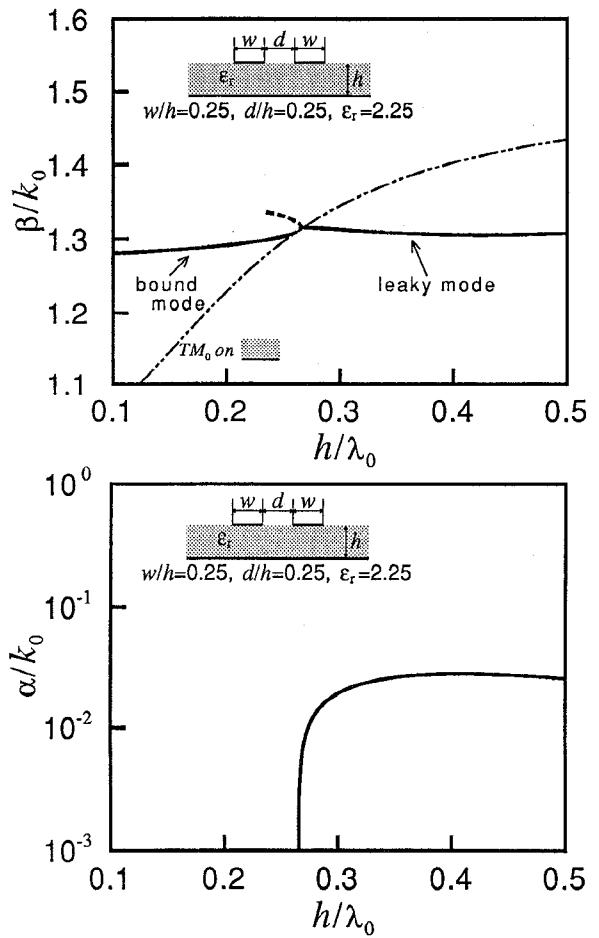


Fig.2. Behaviors of the normalized phase and leakage constants as a function of normalized frequency for conductor-backed coplanar strips when the strips are fairly narrow ( $w/h=0.25$ ). The behaviors for this case are the usual ones.

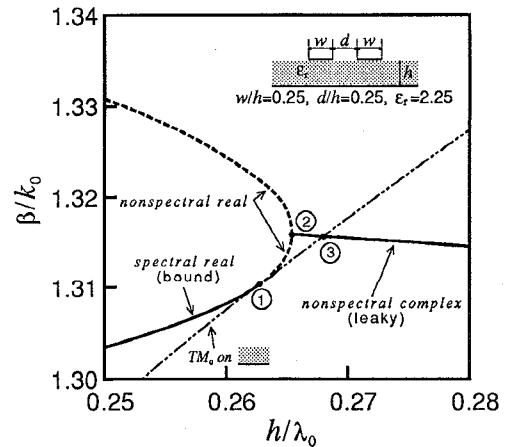


Fig.3. Expanded plot of the spectral gap region for the case considered in Fig.2. This spectral gap is the usual, or regular, type.

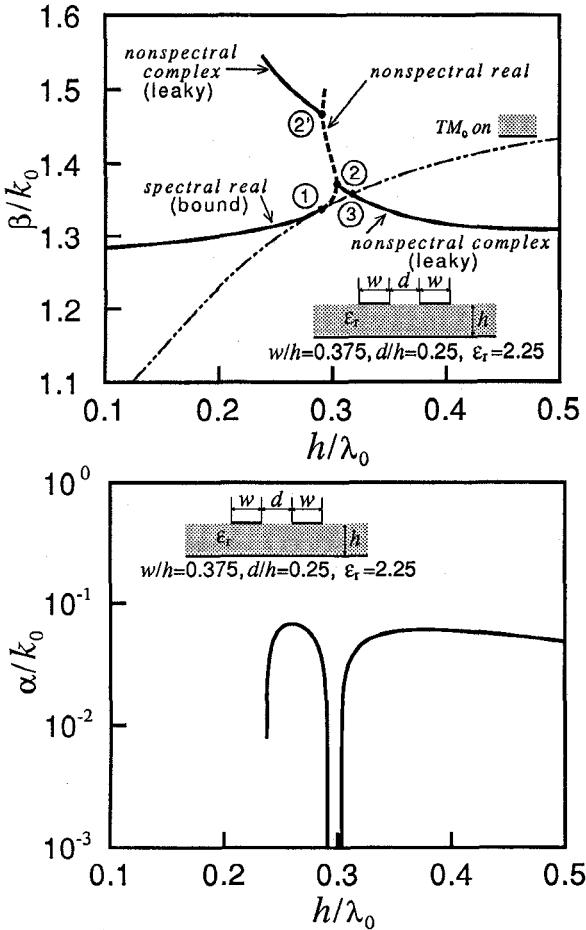


Fig.4(a). Behaviors of the normalized phase and leakage constants as a function of normalized frequency for conductor-backed coplanar strips when the strips are slightly wider ( $w/h = 0.375$ ) than those in Fig.2. The spectral gap now has a new feature: a new leaky, but nonphysical, solution emerging from point ②.

the original leaky solution seen in Fig.3. For that case a spectral gap is still present. As the strip width is increased somewhat more, the leaky solution remains continuous and moves toward point ① in Fig.3, and then goes down past it and below it, so that the leaky solution crosses the real solution at a frequency lower than that corresponding to point ①, and so that the spectral gap disappears. That situation is illustrated in Fig.4(b), where  $w/h$  has been changed to 0.50.

Two important critical frequencies are indicated in Fig.4(b):  $f_{\text{cr1}}$ , the frequency at which the physical leaky dominant mode begins, and  $f_{\text{cr2}}$ , the frequency at which the conventional bound dominant mode ends. In the frequency range between  $f_{\text{cr1}}$  and  $f_{\text{cr2}}$  both the bound and leaky dominant modes are present *simultaneously*.

This overlap region, in which both modes exist simultaneously, becomes *very wide* as the strips are made still wider. For example, when  $w/h$  is increased only a bit more, to 0.70, the value of  $f_{\text{cr2}}$  becomes more than twice that of  $f_{\text{cr1}}$ .

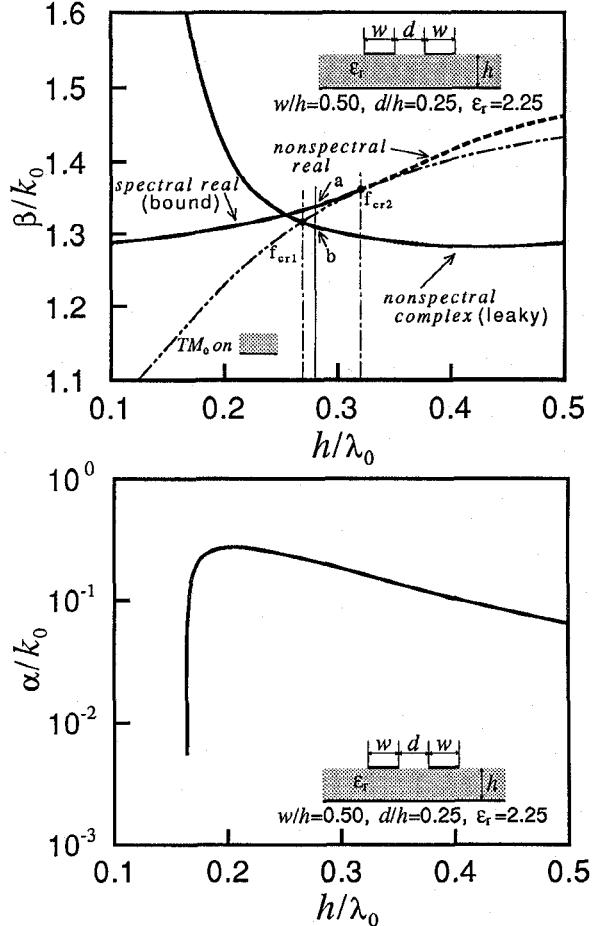


Fig.4(b). Same as Fig.4(a) except that the strips are now somewhat wider ( $w/h = 0.50$ ). The spectral gap has disappeared, and instead there is an overlap region in frequency in which the bound and leaky dominant modes are both present simultaneously.

Curves to verify this statement will be presented in the talk.

We computed the electric field vector behavior to understand more fully the wave behavior in the overlap frequency range between the two critical frequencies  $f_{\text{cr1}}$  and  $f_{\text{cr2}}$ . Those vector plots, made at points *a* and *b* in Fig.4(b), verified the conclusions presented there. They show that for both modes the field is strong in the neighborhood of the central region of the guide, but that for the bound mode it decays in the transverse (*x*) direction, while, for the leaky mode, it increases transversely. These curves are not shown here, but will be presented during the talk.

### III. MEASUREMENTS

Measurements were performed to confirm the new behavioral features discussed above in Sec.II. Two structures were measured; the first one corresponds to the theoretical curves in Fig.2, for which the bound and leaky modes appear in separate frequency ranges, as is customary, and the second one

corresponds to the theoretical curves in Fig.4(b), where the bound and leaky modes are present simultaneously in the frequency range between  $f_{\text{cr1}}$  and  $f_{\text{cr2}}$ . The dimensions for the first structure were  $w=2.08$  mm,  $d=2.08$  mm,  $h=8.30$  mm and  $\epsilon_r=2.25$ ; those for the second structure differed only in the value of  $w$ , which was 4.15 mm. Frequencies  $f_{\text{cr1}}$  and  $f_{\text{cr2}}$  correspond to 9.72 GHz and 11.57 GHz, respectively.

Measurements were made of  $\beta$  for both the bound and leaky modes on both structures, and particular attention was paid to the overlap region for the second structure in which both modes are present simultaneously. In those frequency ranges in which the bound and leaky modes exist separately, the  $\beta$  values are found in a straightforward way. For the **bound** modes, the line was terminated in a short circuit and the value of  $\beta$  was obtained directly from a measurement of the guide wavelength of the standing wave produced along the guide axis. The values of  $\beta$  for the **leaky** modes were obtained by measuring the angle between the guide axis and the leaky wave direction. In the transverse direction, the field of the leaky mode first increases and then decreases. The peak of this field at different distances along the guide forms an angle whose value is directly related to the  $\beta$  of the leaky mode.

In the overlap frequency region for the second structure, where the bound and leaky modes exist simultaneously, the measurements are more difficult to make because the field distributions of the bound and leaky modes are rather similar in the central guiding region. Therefore, a conventional source will excite both modes with roughly equal amplitudes, and both modes will of course be present simultaneously in this overlap frequency range. However, it is fortunately possible to make the required measurements because of the following. The leakage constant of the leaky mode is rather high, as seen from Fig.4(b), corresponding to about 5 dB per wavelength. Thus, the leaky mode decays rather quickly, and after some distance the field on the guide axis is primarily that of the bound mode. The modes can therefore be separated, and their  $\beta$  values were measured independently in the fashion described above. The values of  $\alpha$  for the leaky modes were also measured, and those results will also be presented in the talk but are omitted here due to limited available space.

The measured values were then normalized in the form used in Fig.2 or Fig.4(b), and superimposed on the theoretical plots to obtain direct comparisons, shown in Fig.5 for the first structure and in Fig.6 for the second structure. The measured values are indicated by the white circles, while the solid curves represent the theoretical values already found in Figs.2 and 4(b). The nonphysical theoretical solutions that appear in Figs.2 and 4(b) are omitted here. Although there is a small amount of scatter in the measured points, it is clear that the measurements prove that there is a distinct difference in the  $\beta$  behavior between the two structures, and that both the bound and the leaky modes do appear simultaneously in Fig.6. These measurements indeed verify the new theoretical results reported here.

## ACKNOWLEDGMENT

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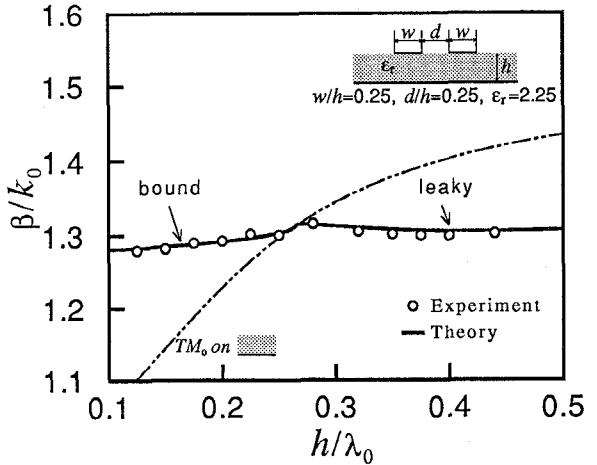


Fig.5. Comparison between measured and theoretical values of normalized phase constant over a wide frequency range, where the white circles indicate the measured values and the solid curves represent the theoretical values already found in Fig.2 in which the spectral gap form is regular.

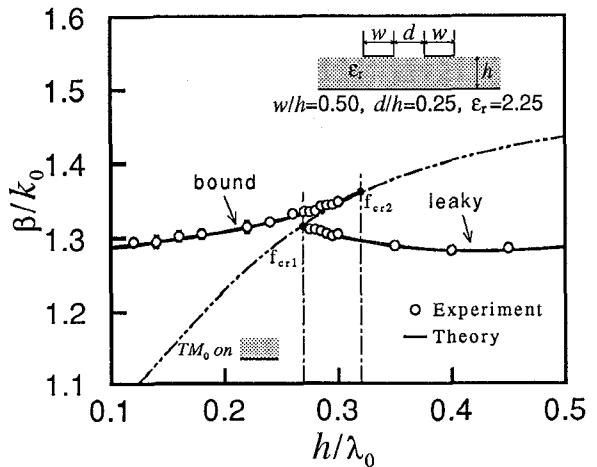


Fig.6. Comparison between measured and theoretical values of normalized phase constant over a wide frequency range, where the white circles indicate the measured values and the solid curves represent the theoretical values already found in Fig.4(b). The measurements clearly confirm the theory, and verify that the bound and leaky dominant modes are present simultaneously in the overlap frequency range.

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